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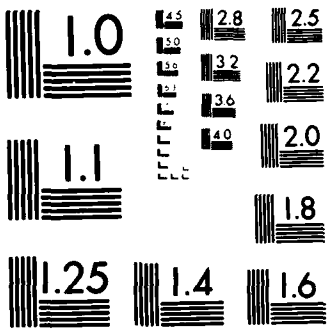
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FINAL REPORT

for the

U. S. ARMY RESEARCH OFFICE

by

ROBERT J. PLEMMONS

on

Grant No. DAAG29-77-G-0166, Research in Numerical Linear Algebra, 15 June 1977 - 14 June 1980.

and

Grant Nos. DAAG29-80-K-0025 and DAAG29-81-K-0132, Numerical Methods for Large-Scale Least Squares and Other Problems, 15 June 1980 - 14 June 1983.

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DEPARTMENTS OF MATHEMATICS AND COMPUTER SCIENCE
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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

HYPERBOLIC SYSTEMS OF PDE, GEODETIC ADJUSTMENTS, M-MATRICES, LEAST SQUARES COMPUTATIONS, SPARSE MATRICES, STATIONARY DISTRIBUTIONS OF MARKOV CHAINS, STRUCTURAL OPTIMIZATION.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report contains a summary of the major accomplishments under the ARO sponsored projects, "Research in Numerical Linear Algebra" and "Numerical Methods for Large-Scale Least Squares and Other Problems", over the six year period 1977-1983. A list of 28 research publications resulting from this work is also included.

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I. THE PROBLEMS STUDIED

The research problems studied for the U.S. Army Research Office over the six year period of these grants are each in the research area of Numerical Linear Algebra and Applications. The objectives of the research were to develop new techniques for handling large-scale problems which involve iterative or direct methods for solving large sparse systems of linear equations or least squares problems. The topics to be studied included: (1) the convergence of iterative methods for solving systems of linear equations, (2) hyperbolic systems of partial differential equations, (3) large-scale least squares computations, (4) M-matrices and related topics on nonnegativity, (5) the computation of stationary distributions for Markov chains and (6) sparse matrix computations in structural optimization.

Research on these topics was to be coordinated with associated scientists and engineers from some of the application areas such as queueing network analysis, compartmental analysis of biological systems, geodetic adjustment methods and structural analysis. Some theoretical research was to be done and certain algorithms and software were to be developed and tested on real problems from these areas.

II. SUMMARY OF MAJOR RESULTS

The most important research accomplishments by the Principal Investigator and co-workers are described below. These results have been obtained in six areas of Numerical Linear Algebra and Applications.

1. Convergence of Iterative Methods for Linear Systems

Given a system of linear equations $Ax = b$, consider the convergence of the iterations

$$x^{(k+1)} = Hx^{(k)} + d, \quad k = 0,1, \dots ,$$

where H is an iteration matrix from some splitting of A, that is, $H = M^{-1}N$, with $A = M - N$. In joint work with James Ortega, the famous Ostrowski-Reich theorem for the convergence of the iterations, where A is hermitian positive definite, has been extended to the case where A is hermitian but not necessarily positive definite and secondly to the case where A is not hermitian but $A + A^*$ is positive definite. In another direction, the convergence of the iterations where A is singular and $A = M - N$ is a regular splitting has been investigated in joint work with C. Meyer and M. Neumann. Three papers have been written on this project. In particular, the paper, "Convergent powers of nonnegative matrices and iterative methods for linear systems", published in 1978, contains results that have been crutial to recent work on matrix methods for queueing networks by Linda Kaufman and others at Bell Laboratories.

2. Hyperbolic systems of Partial Differential Equations

In this project, with M. Gunzburger, the open problem of finding an energy conserving norm for the solution of a hyperbolic system $\frac{\partial u}{\partial t} = A \frac{\partial u}{\partial x}$, subject to boundary conditions, was reduced to that of characterizing certain matrices which can be simultaneously scaled to orthogonal matrices. Subsequently, an efficient althorithm for such scalings was developed in work with A. Berman and B. Parlett. Two papers were written on this topic and the results were presented at a conference on numerical analysis in Hungary.

3. Large-Scale Least Squares Computations

This has probably been the major focal point of research by the Principal Investigator over the past six years and some significant accomplishments have been obtained in joint work with others. The primary application area has been in the least squares adjustment of geodetic data and we have worked closely with personnel from the Defense Mapping Agency (DMA) and the National Geodetic Survey (NGS). The NGS is currently undertaking the readjustment of the North American Datum. In particular, trips were made to these agencies in Washington to discuss mutual problems and to exchange ideas and data. Algorithms and software for solving large-scale least squares problems was written and tested and parts of it are currently being used at the DMA and the NGS. This represents, in part, joint work with A. George, G. Golub and M. Heath. A total of six papers have been written on this topic and several lectures have been given, including colliquium talks at Stanford University, Auburn University, N. C. State University, the DMA and the NGS.

4. M-Matrices and Related Topics on Nonnegativity

The study of M-matrices and their properties involving nonnegativity have elegant and extensive applications in the mathematical sciences. Our work in this area has involved characterizations of such matrices, Lyapunov stability theory for differential equations, convergent regular splittings of M-matrices, triangular LU factorizations and backward error analysis for linear systems associated with inverses of M-matrices. Some of this work, done jointly with R. Funderlic and M. Neumann was presented at the Gatlinburg Symposium held at Oxford, England in 1981. In addition, a research book on this topic, Nonnegative Matrices In the Mathematical Sciences, Academic Press series on Applied Mathematics and Computer Science, was published along with eight other publications relating to M-matrices and applications.

5. Computation of Stationary Distributions of Markov Chains

Consider an irreducible $n \times n$ matrix $A = (a_{ij})$ with $a_{ij} \leq 0$ for all $i \neq j$ and with $\sum_{i=1}^{n} a_{ij} = 0$, $1 \leq j \leq n$. These matrices are called Q-matrices and they arise in several areas, including the analysis of queueing networks from whence the name arises, the analysis of biological compartmental models and the input-output analysis of economic models. The solution $p > 0$ to $Ax = 0$, where $\sum_{i=1}^{n} p_i = 1$, is the stationary distribution vector associated with the underlying Markov chain for the model. Work here, with R. Funderlic and W. Harrod, has concentrated on direct and combined direct-iterative algorithms for computing p and on updating LU factorizations of A. These algorithms have been implemented and they compare quite favorably with current methods for computing p. Three papers were written to report these results.

6. Sparse Matrix Computations in Structural Optimization

A very promising and important application of sparse matrix technology is that of developing algorithms and software for problems in structural analysis and optimal design. Given the external loads on a structure, the object of structural analysis is to determine the resulting internal forces, stresses and displacements. This involves solving a quadratic programming problem,

$$\min_{F} \; F^T fF \quad \text{such that } EF = P,$$

for the internal force vector F, where f is the element flexibility matrix, E is the equilibrium matrix and P is the external load vector. The Principal Investigator, along with joint authors M. Berry, M. Heath, K. Kaneko and R. Ward, have recently been involved with developing algorithms and software for using the force method in structural optimization. All of this software was written at N. C. State University. A primary feature of our work is the development of an efficient algorithm for computing a sparse and banded basis of the null space of E to remove the redundant forces. The work is incomplete here, but a research grant has been made by the Air Force Office of Scientific research to enable the Principal Investigator to continue this project. Four papers have been written thus far on this research and results have been presented at the Sparse Matrix Symposium - 1982, at the First Army Conference on Applied Mathematics and Computing and at the Mathematics Research Center, University of Wisconsin.

To summarize, the Principal Investigator feels that very many of the research objectives of these two grants have been met and that the overall results will provide useful tools for scientists and engineers inside and outside the Department of Defense.

III. TECHNICAL PUBLICATIONS

1. "Convergent powers of a matrix with applications to iterative methods for singular linear systems," SIAM J. Numer. Analy., 14(1977), 699-705, with C. Meyer.

2. "M-matrix characterizations I: nonsingular M-matrices," Linear Algebra and Its Applications, 18(1977), 175-188.

3. "Convergent powers of nonnegative matrixes and iterative methods for linear systems," Numer. Math., 31(1978), 265-279, with M. Neumann.

4. "Positive diagonal solutions to the Lyapunov equation," Linear and Multilinear Algebra, 5(1978), 249-256, with G. Barker and A. Berman.

5. "Generalized inverse positivity and splittings of M-matrices," Linear Algebra and Its Applications, 23(1979), 21-35, with M. Neumann.

6. "Energy conserving norms for hyperbolic systems of PDE," Mathematics of Comp., 33(1979), 1-10, with Max Gunzburger.

7. "Extensions of the Ostrowski-Reich Theorem for SOR iterations," special issue of Lin. Alg. and Its Appl., 28(1979), 177-192, dedicated to Alston Householder's 75th Birthday, with J. M. Ortega.

8. "Adjustment by least squares in geodesy using block iterative methods for sparse matrices," January 1979, Proc. U.S. Army Conf. on Numer. Anal. and Computers, 151-185.

9. "M-matrix characterizations II: general M-matrices," Linear and Multilinear Alg., 9(1980), 211-255, with M. Neumann.

10. "Large scale geodetic least squares adjustment by dissection and orthogonal decomposition," Linear Algebra and Its Appl., 34(1980), 3-28, with G. H. Golub.

11. "Sparse least squares problems," Proc. 1979 International Sympsoium on Computing Methods in Applied Science and Eng., Versailles, France, with G. H. Golub.

12. "Diagonal scaling to an orthogonal matrix," SIAM J. on Algebraic and Discrete Methods, 2(1981), 57-65, with A. Berman and B.Parlett.

13. "Software for ordering sparse least square problems prior to Givens reductions," Proc. U.S. Army Conf. on Numer. Anal. and Computers, Huntsville, Alabama, 1981, with D. Hume and J. Litsey.

14. "Solution of large scale sparse least squares problems using auxillary storage," SIAM J. Sci. and Stat. Computing, 2(1981), 416-429, with A. George and M. T. Heath.

15. "LU decomposition of M-matrices by elimination without pivoting," Linear Algebra and Applications, 41(1981), 99-119, with R. Funderlic.

16. "Least squares adjustment of large-scale geodetic networks by sparse orthogonal decomposition," Proc. of International Symposium on Geodetic Networks and Computations, Munich, Germany, 1981, with A. George, G. Golub and M. Heath.

17. "Stability of LU decompositions of generalized diagonally dominant matrices," Numerische Mathematik, 40(1982), 57-69, with R. Funderlic and M. Neumann.

18. "Comparison of some direct methods for computing stationary distributions of Markov chains," SIAM J. on Sci. and Stat. Computing, (to appear), with W. Harrod.

19. "Incomplete factorization methods for certain M-matrices", SIAM J. on Algebraic and Discrete Methods, (to appear), with R. Funderlic.

20. "Minimum norm solutions to linear elastic analysis problems", Int. J. Numerical Methods in Engineering (to appear) with I. Kaneko.

21. "Orthogonal schemes for structural optimization", Proceedings of the First Army Conference on Applied Math. and Computing, (to appear) with M. Berry, M. Heath and R. Ward.

22. "Backward error analysis for linear systems associated with inverses of H-matrices, submitted to BIT, with M. Neumann.

23. "Sparse orthogonal schemes for structural optimization using the force method", submitted to SIAM J. on Sci. and Stat. Computing, with M. Heath and R. Ward.

24. "Convergence of Gauss-Seidel iterations for computing stationary distributions of Markov Chains, submitted to the SIAM J. on Alg. and Dis. Methods, with G. Barker.

Manuscripts in Preparation

"Algorithms and software for computing a banded basis of the null space", with M. Berry, M. Heath, I Kaneko, M. Lawo and R. Ward.

"Updating methods for computing stationary distributions", with R. Funderlic.

Books

Nonnegative Matrices in the Mathematical Sciences, Academic Press, N.Y., 1979, 312 pp., with A. Berman.

Large Scale Matrix Problems, North Holland, NY, 1981, 396 pp., with A. Björck and H. Schneider (editors).

IV. PARTICIPATING SCIENTIFIC PERSONNEL

The only scientific personnel supported by funds from the U. S. Army Research Office under these grants was the Principal Investigator, R. J. Plemmons. However, one Ph.D. student and seven Masters Degree Students were directed during this period. A list of their names and theses topics follows.

W. Harrod, "Rank Modification Methods for Certain Linear Systems", Ph.D., Fall 1982 (now an Assistant Professor at the University of Kentucky).

E. Lagarre, "Classes of Matrices Determined by Certain Eigenvalue Properties", M.S., Fall 1978.

G. Melendez, "The Large Sparse Linear Least Squares Problem", M.S., Fall 1978.

C. Joyner, "Block Decomposition Algorithms for Matrices in Dual Angular Form", M.S., Fall 1980.

J. Smith, "The Natural Factor Formulation of the Finite Element Problem", M.S., Fall 1980.

J. Litsey, "Finding a Block Upper Trapezoidal Form of a Rectangular Matrix", M.S., Fall 1981.

D. Hume, "Algorithms and Software for Ordering Sparse Least Squares Problems Prior to Givens Reduction", M.S., Fall 1981.

M. Berry, "Algorithms and Software for the Force Method in Structural Analysis", M.S., Spring 1983.

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